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Question Booklet Version

22

Aakash

Medical | IIT-JEE | Foundations

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Time : 1 Hour 30 Min.

Total Marks : 100

Answers & Solutions for MHT CET-2018

Paper-I (Mathematics)

Instruction for Candidates

1. This question booklet contains 50 Objective Type Questions (Single Best Response Type) in the subject of Mathematics (50).
2. The question paper and OMR (Optical Mark Reader) Answer Sheet are issued to examinees separately at the beginning of the examination session.
3. Choice and sequence for attempting questions will be as per the convenience of the candidate.
4. Read each question carefully.
5. Determine the correct answer from out of the four available options given for each question.
6. Each answer with correct response shall be awarded **two (2) marks**. There is **no Negative Marking**. If the examinee has marked two or more answers or has done scratching and overwriting in the Answer Sheet in response to any question, or has marked the circles inappropriately *e.g.*, half circle, dot, tick mark, cross etc. mark/s shall NOT be awarded for such answer/s, as these may not be read by the scanner. Answer sheet of each candidate will be evaluated by computerized scanning method only (Optical Mark Reader) and there will not be any manual checking during evaluation or verification.
7. Rough work should be done only on the blank space provided in the Question Booklet. **Rough work should not be done on the Answer Sheet**.
8. The required mathematical tables (Log etc.) are provided within the Question Booklet.

MATHEMATICS

1. If $\int_0^K \frac{dx}{2+18x^2} = \frac{\pi}{24}$, then the value of K is

(A) 3 (B) 4

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Answer (C)

Sol. $\int_0^K \frac{dx}{2+18x^2} = \frac{1}{2}\int_0^K \frac{dx}{(3x)^2+1}$

$$= \frac{1}{6}\tan^{-1}3x\Big]_0^K$$

$$= \frac{1}{6}\tan^{-1}3K = \frac{\pi}{24}$$

hence $K = \frac{1}{3}\tan\frac{\pi}{4} = \frac{1}{3}$

2. The cartesian co-ordinates of the point on the parabola $y^2 = -16x$, whose parameter is $\frac{1}{2}$, are

(A) (–2, 4) (B) (4, –1)

(C) (–1, –4) (D) (–1, 4)

Answer (C, D)*

Sol. *For the curve $y^2 = -16x$, the parametric form of coordinates of any point are $(-4t^2, \pm 8t)$. Hence for $t = \frac{1}{2}$, point is $(-1, \pm 4)$.

3. $\int \frac{1}{\sin x.\cos^2 x}dx =$

(A) $\sec x + \log|\sec x + \tan x| + c$

(B) $\sec x.\tan x + c$

(C) $\sec x + \log|\sec x - \tan x| + c$

(D) $\sec x + \log|\operatorname{cosec} x - \cot x| + c$

Answer (D)

Sol. The given integral can be written as

$$\int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x}dx$$

$$= \int(\sec x \tan x + \operatorname{cosec} x)dx$$

$$= \sec x + \log|\operatorname{cosec} x - \cot x| + c$$

4. If $\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right) = 2$ then $\frac{dy}{dx} =$

(A) $\frac{x}{y}$ (B) $-\frac{y}{x}$

(C) $-\frac{x}{y}$ (D) $\frac{y}{x}$

Answer (D)

Sol. Taking antilog, $\frac{x^3-y^3}{x^3+y^3} = 10^2$.

Applying componendo and dividendo,

$$\frac{(x^3-y^3)+(x^3+y^3)}{(x^3-y^3)-(x^3+y^3)} = \frac{100+1}{100-1}$$

Simplifying, we have $y^3 = \frac{-99x^3}{101}$.

Differentiating,

$$3y^2\frac{dy}{dx} = \frac{-99\times 3x^2}{101}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{99}{101}\frac{x^2}{y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^3}{x^3}\times\frac{x^2}{y^2} = \frac{y}{x}$$

5. If $f : R - \{2\} \to R$ is a function defined by $f(x) = \frac{x^2-4}{x-2}$, then its range is

(A) R (B) $R - \{2\}$

(C) $R - \{4\}$ (D) $R - \{-2, 2\}$

Answer (C)

Sol. The function can be simplified as $y = x+2, x \neq 2$. The value corresponding to $x = 2$ is not in the range. Hence Range is $R - \{4\}$

6. If $f(x) = x^2 + \alpha$ for $x \geq 0$

$$= 2\sqrt{x^2+1} + \beta \text{ for } x < 0$$

is continuous at $x = 0$ and $f\left(\frac{1}{2}\right) = 2$ then $\alpha^2 + \beta^2$ is

(A) 3 (B) $\frac{8}{25}$

(C) $\frac{25}{8}$ (D) $\frac{1}{3}$

Answer (C)

Sol. Since the function $f(x)$ is continuous, we must have

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

But $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2\sqrt{x^2+1}+\beta = 2+\beta,\ f(0)=\alpha$

and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2+\alpha = \alpha$

Hence $2 + \beta = \alpha$. ....(i)

$$f\left(\frac{1}{2}\right) = \frac{1}{4}+\alpha = 2 \Rightarrow \alpha = \frac{7}{4}, \beta = -\frac{1}{4}$$

$$\Rightarrow \alpha^2+\beta^2 = \frac{25}{8}$$

7. If $y = \left(\tan^{-1}x\right)^2$ then

$$\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx} =$$

(A) 4 (B) 2

(C) 1 (D) 0

Answer (B)

Sol. Differentiating the given function we get $\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}$. Differentiating second time, we get

$\frac{d^2y}{dx^2} = \frac{2-4x\tan^{-1}x}{\left(1+x^2\right)^2}$. Rearranging the last equation we have, $\left(1+x^2\right)^2\frac{d^2y}{dx^2} = 2-4x\tan^{-1}x$.

Substituting $\tan^{-1}x = \frac{(1+x^2)}{2}\frac{dy}{dx}$, we finally get

$$(1+x^2)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx} = 2$$

8. The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - kx - 2y + 2 = 0$ then the value of k is

(A) –11 (B) 31

(C) 11 (D) –31

Answer (C)

Sol. If the lines represented by pair of lines $ax^2 + by^2 + 2gx + 2fy + 2hxy + c = 0$ has slopes m_1 and m_2, then we have $m_1 + m_2 = -\frac{2h}{b}$, which in this case is not defined as $b = 0$.Since slope of one of the lines is given as $m = -5$, the slope of other must be infinite. Hence the other line must be of the form $x = k_1$. Comparing term by term $(5x + y - 1)(x - k_1) \equiv 5x^2 + xy - kx - 2y + 2$, we get $k = 11$

9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ then $\left(A^2-5A\right)A^{-1} =$

(A) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -4 & -1 & 1 \\ 2 & -4 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & -2 & 1 \\ 4 & -2 & -3 \\ 1 & 4 & -2 \end{bmatrix}$

Answer (B)

Sol. $\left(A^2-5A\right)A^{-1} = A-5I$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

10. The equation of line passing through (3, –1, 2) and perpendicular to the lines

$\bar{r} = \left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ and

$\bar{r} = \left(2\hat{i}+\hat{j}-3\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ is

(A) $\frac{x+3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(B) $\frac{x-3}{3} = \frac{y+1}{2} = \frac{z-2}{2}$

(C) $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(D) $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-2}{3}$

Answer (C)

Sol. The direction vector of the line is given by

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -2\hat{i}-3\hat{j}-2\hat{k}$. Since line passes through (3, –1, 2), its equation is

$$\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$$

11. Letters in the word **HULULULU** are rearranged. The probability of all three **L** being together is

(A) $\frac{3}{20}$ (B) $\frac{2}{5}$

(C) $\frac{3}{28}$ (D) $\frac{5}{23}$

Answer (C)

Sol. size of the sample space, $n(S) = \frac{8!}{4!\,3!}$

number of favorable cases, $n(E) = \frac{6!}{4!}$

$$P = \frac{n(E)}{n(S)} = \frac{6!\,4!\,3!}{4!\,8!} = \frac{6}{8\times 7} = \frac{3}{28}$$

12. The sum of the first 10 terms of the series $9+99+999+.....$ is

(A) $\frac{9}{8}\left(9^{10}-1\right)$ (B) $\frac{100}{9}\left(10^{9}-1\right)$

(C) 10^9-1 (D) $\frac{100}{9}\left(10^{10}-1\right)$

Answer (B)

Sol. $9 + 99 + 999 + \cdots$

$[(10-1) + (10^2-1) + (10^3-1) + \cdots 10^{th}\text{ term}]$

$[(10^1 + 10^2 + 10^3 + \cdots 10^{th}\text{ term}) - 10]$

$$=\left[\frac{10\left[1-10^{10}\right]}{1-10}-10\right]$$

$$=\left[10\frac{\left(10^{10}-1\right)}{9}-10\right]=\frac{10}{9}\left[\left(10^{10}-1\right)-9\right]$$

$$=\frac{100}{9}\left[10^9-1\right]$$

13. If A, B, C are the angles of ΔABC then $\cot A.\cot B + \cot B.\cot C + \cot C.\cot A =$

(A) 0 (B) 1

(C) 2 (D) –1

Answer (B)

Sol. Since $\tan(A+B+C) = \tan\pi = 0$.

$$\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}=0$$

Thus

$$\tan A+\tan B+\tan C-\tan A\tan B\tan C=0$$

Dividing by $\tan A \tan B \tan C$ and simplifying we get

$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

14. If $\int \frac{dx}{\sqrt{16-9x^2}} = A\sin^{-1}(Bx)+C$ then $A + B =$

(A) $\frac{9}{4}$ (B) $\frac{19}{4}$

(C) $\frac{3}{4}$ (D) $\frac{13}{12}$

Answer (D)

Sol. $$\int\frac{dx}{\sqrt{16-9x^2}}=\frac{1}{3}\int\frac{dx}{\sqrt{\frac{16}{9}-x^2}}=\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$$

Comparing, we get $A = \frac{1}{3}$, $B = \frac{3}{4}$

Therefore $A + B = \frac{1}{3}+\frac{3}{4}=\frac{4+9}{12}=\frac{13}{12}$

15. $\int e^x\left[\frac{2+\sin 2x}{1+\cos 2x}\right]dx =$

(A) $e^x\tan x+c$ (B) $e^x+\tan x+c$

(C) $2e^x\tan x+c$ (D) $e^x\tan 2x+c$

Answer (A)

Sol. $$\int e^x\left[\frac{2+\sin(2x)}{1+\cos 2x}\right]dx$$

$$=\int e^x\left[\frac{2+2\sin x\cos x}{2\cos^2 x}\right]dx$$

$$=\int e^x\left[\sec^2 x+\tan x\right]dx$$

$= e^x\tan x + c$

16. A coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails then $P(X = 1) =$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{3}{4}$

Answer (D)

Sol. There are two possible favourable cases: *HHT* or *TTH*. Where *H* denotes Head and *T* denotes Tail.

$$P(X = 1) = {}^3C_1 \times \left(\frac{1}{2}\right)^3 \times 2 = \frac{3}{4}$$

17. If $2\sin\left(\theta+\frac{\pi}{3}\right)=\cos\left(\theta-\frac{\pi}{6}\right)$ then $\tan\theta$ =

(A) $\sqrt{3}$ (B) $-\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{3}}$ (D) $-\sqrt{3}$

Answer (D)

Sol. $2\sin\left(\theta+\frac{\pi}{3}\right)=\cos\left(\theta-\frac{\pi}{6}\right)$

$$2\left[\frac{\sin\theta}{2}+\frac{\sqrt{3}\cos\theta}{2}\right]=\frac{\sqrt{3}\cos\theta}{2}+\frac{\sin\theta}{2}$$

$$2\sin\theta+2\sqrt{3}\cos\theta=\sqrt{3}\cos\theta+\sin\theta$$

$$\sin\theta=-\sqrt{3}\cos\theta$$

$$\tan\theta = -\sqrt{3}$$

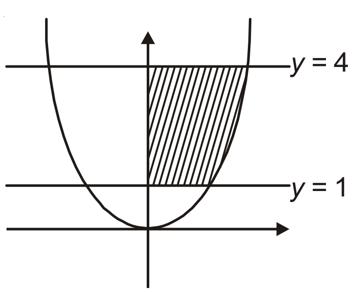
18. The area of the region bounded by $x^2 = 4y$, $y = 1$, $y = 4$ and the y-axis lying in the first quadrant is ________ square units.

(A) $\frac{22}{3}$ (B) $\frac{28}{3}$

(C) 30 (D) $\frac{21}{4}$

Answer (B)

Sol. $y = \frac{x^2}{4}$



Required area = $\int_1^4 2\sqrt{y}\,dy = \frac{2}{3}\cdot 2y^{3/2}\Big]_1^4 = \frac{28}{3}$

19. If $f(x)=\frac{e^{x^2}-\cos x}{x^2}$, for $x \neq 0$ is continuous at $x = 0$, then value of $f(0)$ is

(A) $\frac{2}{3}$ (B) $\frac{5}{2}$

(C) 1 (D) $\frac{3}{2}$

Answer (D)

Sol. Apply L'Hospital Rule,

$$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}=\lim_{x\to 0}\frac{2xe^{x^2}+\sin x}{2x}$$

$$\lim_{x\to 0} e^{x^2}+\frac{\sin x}{2x}=1+\frac{1}{2}=\frac{3}{2}$$

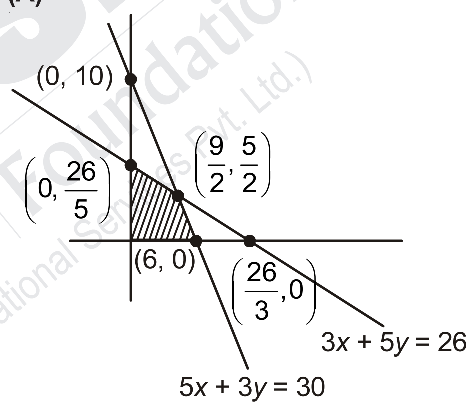
20. The maximum value of $2x + y$ subject to $3x + 5y \le 26$ and $5x + 3y \le 30$, $x \ge 0$, $y \ge 0$ is

(A) 12 (B) 11.5

(C) 10 (D) 17.33

Answer (A)

Sol.



Evaluating the function $2x + y$ at the corner points of the shaded region, it can easily be seen that the maximum occurs at the point (6,0) and is 12.

21. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors having magnitudes 1, 2, 3 respectively, then

$\left[\vec{a}+\vec{b}+\vec{c}\ \ \vec{b}-\vec{a}\ \ \vec{c}\right]$ =

(A) 0 (B) 6

(C) 12 (D) 18

Answer (C)

Sol. $\left[\vec{a}+\vec{b}+\vec{c}\quad \vec{b}-\vec{a}\quad \vec{c}\right]$

$= (\vec{a}+\vec{b}+\vec{c})\cdot((\vec{b}-\vec{a})\times\vec{c})$

$= (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{b}\times\vec{c}-\vec{a}\times\vec{c})$

$= [\vec{a}\ \vec{b}\ \vec{c}] - [\vec{b}\ \vec{a}\ \vec{c}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

$= 2\times 1\times 2\times 3 = 12$

22. If points $P(4, 5, x)$, $Q(3, y, 4)$ and $R(5, 8, 0)$ are collinear, then the value of $x + y$ is

(A) –4 (B) 3

(C) 5 (D) 4

Answer (D)

Sol. $\frac{4-3}{3-5} = \frac{5-y}{y-8} = \frac{x-4}{4}$

$-\frac{1}{2} = \frac{5-y}{y-8}$

$y - 8 = -10 + 2y \quad \Rightarrow \quad y = 2$

$-\frac{1}{2} = \frac{x-4}{4}$

$-2 = x - 4 \quad \Rightarrow \quad x = 2$

$x + y = 2 + 2 = 4$

23. If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is two times the other then

(A) $8h^2 = 9ab$

(B) $8h^2 = 9ab^2$

(C) $8h = 9ab$

(D) $8h = 9ab^2$

Answer (A)

Sol. Let the slopes of lines be m, $2m$

sum of roots $m + 2m = \frac{-2h}{b}$

$\Rightarrow 3m = \frac{-2h}{b}$ ...(i)

product of roots $2m^2 = \frac{a}{b}$ ...(ii)

eliminating m from both equations we get

$8h^2 = 9ab$

24. The equation of the line passing through the point (–3, 1) and bisecting the angle between co-ordinate axes is

(A) $x + y + 2 = 0$

(B) $-x + y + 2 = 0$

(C) $x - y + 4 = 0$

(D) $2x + y + 5 = 0$

Answer **

Sol. **Wrong Question

The lines bisecting the angle between coordinate axes are $y = x$ and $y = -x$ which do not passes through the point (–3, 1)

25. The negation of the statement : "Getting above 95% marks is necessary condition for Hema to get the admission in good college".

(A) Hema gets above 95% marks but she does not get the admission in good college

(B) Hema does not get above 95% marks and she gets admission in good college

(C) If Hema does not get above 95% marks then she will not get the admission in good college

(D) Hema does not get above 95% marks or she gets the admission in good college

Answer (B)

Sol. Let A denote "Hema to get the admission in good college"

B denote "Getting above 95% marks"

Then given statement is $A \to B$. Negation of this statement is

$\sim(A \to B) = \sim(\sim A \vee B) = A \wedge \sim B$

Hence the required negation in words is "Hema does not get above 95% marks and she gets admission in good college".

26. $\cos 1° \cdot \cos 2° \cdot \cos 3° \cdot \cdot \cdot \cos 179° =$

(A) 0 (B) 1

(C) $-\frac{1}{2}$ (D) –1

Answer (A)

Sol. $\cos 90° = 0$

27. If planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line then $a^2 + b^2 + c^2 =$

(A) $1 - abc$ (B) $abc - 1$

(C) $1 - 2abc$ (D) $2abc - 1$

Answer (C)

Sol. $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - c^2 - abc - abc - b^2 = 0$

$a^2 + b^2 + c^2 = 1 - 2abc$

28. The point of intersection of lines represented by $x^2 - y^2 + x + 3y - 2 = 0$ is :

(A) (1, 0) (B) (0, 2)

(C) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Answer (C)

Sol. Let $s = x^2 - y^2 + x + 3y - 2$

$$\frac{ds}{dx} = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

$$\frac{ds}{dy} = -2y + 3 = 0 \Rightarrow y = \frac{3}{2}$$

29. A die is rolled. If X denotes the number of positive divisors of the outcome then the range of the random variable X is :

(A) {1, 2, 3}

(B) {1, 2, 3, 4}

(C) {1, 2, 3, 4, 5, 6}

(D) {1, 3, 5}

Answer (B)

Sol.

Outcomes	Divisors	Number of Divisors
1	{1}	1
2	{1,2}	2
3	{1,3}	2
4	{1,2,4}	3
5	{1,5}	2
6	{1,2,3,6}	4

Hence range of X is {1, 2, 3, 4}.

30. A die is thrown four times. The probability of getting perfect square in at least one throw is :

(A) $\frac{16}{81}$ (B) $\frac{65}{81}$

(C) $\frac{23}{81}$ (D) $\frac{58}{81}$

Answer (B)

Sol. Required Probability = 1 – P(No perfect square in any throw)

Perfect square are 1, 4. Probability of perfect square in any one throw = $\frac{2}{6} = \frac{1}{3}$

P(at least one perfect. square)

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{16}{81} = \frac{81-16}{81}$$

$$= \frac{65}{81}$$

31. $\int_0^{\frac{\pi}{4}} x.\sec^2 x.dx =$

(A) $\frac{\pi}{4} + \log\sqrt{2}$ (B) $\frac{\pi}{4} - \log\sqrt{2}$

(C) $1 + \log\sqrt{2}$ (D) $1 - \frac{1}{2}\log 2$

Answer (B)

Sol. Using integration by parts

$$\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx = x\tan x\Big]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x\,dx$$

$$= \left[\frac{\pi}{4} - \log\left(\sqrt{2}\right)\right]$$

32. In ΔABC, with usual notations, if a, b, c are in A.P. then a $\cos^2\left(\frac{C}{2}\right)$ + c $\cos^2\left(\frac{A}{2}\right)$ =

(A) $3\frac{a}{2}$ (B) $3\frac{c}{2}$

(C) $3\frac{b}{2}$ (D) $\frac{3abc}{2}$

Answer (C)

Sol. $\frac{a}{2}[1+\cos C] + \frac{c}{2}(1+\cos A)$

$$= \frac{a}{2} + \frac{c}{2} + \frac{a\cos C + c\cos A}{2}$$

$$\frac{a+b+c}{2} = \frac{3b}{2}$$

33. If $x = e^\theta(\sin\theta - \cos\theta)$, $y = e^\theta(\sin\theta + \cos\theta)$ then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) 1 (B) 0

(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Answer (A)

Sol. $x = e^\theta(\sin\theta - \cos\theta)$

$y = e^\theta(\sin\theta + \cos\theta)$

$$\frac{dy}{dx} = \frac{e^\theta[\cos\theta - \sin\theta] + (\sin\theta + \cos\theta)e^\theta}{e^\theta[\cos\theta + \sin\theta] + (\sin\theta - \cos\theta)e^\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = 1$$

34. The number of solutions of $\sin x + \sin 3x + \sin 5x = 0$ in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is

(A) 2 (B) 3

(C) 4 (D) 5

Answer (B)

Sol. $\sin x + \sin 3x + \sin 5x = 0$

$2\sin 3x \cos 2x + \sin 3x = 0$

$\sin 3x\,[2\cos 2x + 1] = 0$

either $\sin 3x = 0$ or $2\cos 2x + 1 = 0$

case (1), $\sin 3x = 0$, then $3x = n\pi$

$x = \frac{n\pi}{3}$, n is any integer

If solution lies in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, then $n = 2$,

$n = 3$, $n = 4$, $x = \left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$

Case (2), $\cos 2x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

Then $2x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{Z}$

$x = n\pi \pm \frac{\pi}{3}$, which gives two solutions $\frac{2\pi}{3}, \frac{4\pi}{3}$

Hence total 3 solutions exist in the interval.

35. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, then $x =$

(A) –1 (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{2}$

Answer (C)

Sol. $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\tan^{-1}\frac{5x}{1-6x^2} = \frac{\pi}{4}$

$\frac{5x}{1-6x^2} = 1$

$6x^2 + 5x - 1 = 0$

$6x^2 + 6x - x - 1 = 0$

$6x(x + 1) - (x + 1) = 0$

$(6x - 1)(x + 1) = 0$

$x = \frac{1}{6}, x = -1$

But $x = -1$ does not satisfy the given equation.

$\Rightarrow x = \frac{1}{6}$

36. Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ then the value of $a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33}$ is

(A) 1 (B) 13

(C) –1 (D) –13

Answer (C)

Sol. It is ambiguous that is denoted by a_{ij}.

Assuming a_{ij} to be elements of co-factor matrix we have $a_{31} = (-1)^{3+1}(5{\cdot}2 - 1{\cdot}3) = 7$, $a_{32} = (-1)^{3+2}(5{\cdot}1 - 1{\cdot}3) = -2$, $a_{33} = (-1)^{3+3}(1{\cdot}1 - 1{\cdot}2) = -1$. Hence $a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33} = 7{\cdot}2 + (-2{\cdot}4) + (-1{\cdot}7) = -1$.

37. The contrapositive of the statement : "If the weather is fine then my friends will come and we go for a picnic."

(A) The weather is fine but my friends will not come or we do not go for a picnic

(B) If my friends do not come or we do not go for picnic then weather will not be fine

(C) If the weather is not fine then my friends will not come or we do not go for a picnic

(D) The weather is not fine but my friends will come and we go for a picnic

Answer (B)

Sol. Let A denote "Weather is fine"

B denote "my friends will come"

C denote "we go for a picnic"

Then given statement is $A \to (B \wedge C)$. To find its contrapositive, we need to evaluate $\sim(B \wedge C) \to \sim A$ *i.e.*, $(\sim B \vee \sim C) \to \sim A$.

In words this becomes "If my friends do not come or we do not go for picnic then weather will not be fine".

38. If $f(x)=\frac{x}{x^2+1}$ is increasing function then the value of x lies in

(A) R (B) $(-\infty,-1)$

(C) $(1,\infty)$ (D) $(-1, 1)$

Answer (D)

Sol. $f(x) = \frac{x}{x^2+1}$

$$f'(x)=\frac{1\cdot(x^2+1)-x\cdot(2x)}{(x^2+1)^2}>0$$

$1 - x^2 > 0$

$x^2 - 1 < 0$

$x \in (-1, 1)$

39. If $X=(4^n-3n-1:n\in N)$ and $Y=\{9(n-1):n\in N\}$, then $X\cap Y=$

(A) X (B) Y

(C) ϕ (D) $\{0\}$

Answer (A)

Sol. Consider set X, its element are of the form $4^n - 3n - 1$

$= (1 + 3)^n - 3n - 1$

$= \left(1+{}^nC_1 3+{}^nC_2 3^2+\cdots\right)-3n-1$

$= 3^2\left[{}^nC_2+\cdots{}^nC_n 3^{n-2}\right] \equiv 9p$

Set Y consists of all non negative multiples of 9.

Hence every element of set X is a subset of Y.

40. The statement pattern $p\wedge(\sim p\wedge q)$ is

(A) A tautology

(B) A contradiction

(C) Equivalent to $p\wedge q$

(D) Equivalent to $p\vee q$

Answer (B)

Sol. Consider the truth table of $p\wedge(\sim p\wedge q)$

p	q	$\sim p$	$\sim p\wedge q$	$p\wedge(\sim p\wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

Hence a Contradiction.

41. If the line $y = 4x - 5$ touches to the curve $y^2 = ax^3 + b$ at the point (2, 3) then $7a + 2b$ =

(A) 0 (B) 1

(C) –1 (D) 2

Answer (A)

Sol. $y^2 = ax^3 + b$ ...(1)

Since (2,3) lies on the curve, we have

$9 = 8a + b$ ...(2)

Differentiating (1), $2y\frac{dy}{dx}=3ax^2 \Rightarrow \frac{dy}{dx}=\frac{3ax^2}{2y}$

Since line $y = 4x - 5$ touches the curve

$\Rightarrow \frac{3a\times4}{2\times3}=4\Rightarrow a=2\Rightarrow b=-7$

$\therefore\ 7a + 2b = 0$

42. The sides of a rectangle are given by $x = \pm a$ and $y = \pm b$. The equation of the circle passing through the vertices of the rectangle is

(A) $x^2 + y^2 = a^2$

(B) $x^2 + y^2 = a^2 + b^2$

(C) $x^2 + y^2 = a^2 - b^2$

(D) $(x - a)^2 + (y - b)^2 = a^2 + b^2$

Answer (B)

Sol. (a, b) and $(-a, -b)$ will be ends of diameter.

$\therefore$ Required equation of circle is

$(x + a)(x - a) + (y + b)(y - b) = 0$

$\Rightarrow\ x^2 + y^2 = a^2 + b^2$

43. The minimum value of the function $f(x) = x\log x$ is

(A) $-\frac{1}{e}$ (B) $-e$

(C) $\frac{1}{e}$ (D) e

Answer (A)

Sol. $f(x) = x\log x$

$$f'(x)=1+\log_e x\Rightarrow f'(x)=0\Rightarrow x=\frac{1}{e}$$

and $f''(x)=\frac{1}{x}\Rightarrow f''\left(\frac{1}{e}\right)>0\Rightarrow$ minima exists.

Therefore minimum value of $f(x)$ will occur at $x=\frac{1}{e}$

$$\Rightarrow f\left(\frac{1}{e}\right)=\frac{1}{e}\log\frac{1}{e}=-\frac{1}{e}$$

44. If $X \sim B(n, p)$ with $n = 10$, $p = 0.4$ then $E(X^2) =$

(A) 4 (B) 2.4

(C) 3.6 (D) 18.4

Answer (D)

Sol. $Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = Var(X) + (E(X))^2$

$= npq + (np)^2$

$= (10 \times .4 \times .6) + (10 \times .4)^2$

$= 2.4 + 16$

$= 18.4$

45. The general solution of differential equation $\frac{dx}{dy} = \cos(x+y)$ is

(A) $\tan\left(\frac{x+y}{2}\right) = y + c$

(B) $\tan\left(\frac{x+y}{2}\right) = x + c$

(C) $\cot\left(\frac{x+y}{2}\right) = y + c$

(D) $\cot\left(\frac{x+y}{2}\right) = x + c$

Answer (A)

Sol. $\frac{dx}{dy} = \cos(x+y)$

Let $x + y = t$

$\Rightarrow \frac{dx}{dy} + 1 = \frac{dt}{dy}$

$\therefore \frac{dt}{dy} - 1 = \cos t$

$\Rightarrow \frac{dt}{dy} = 1 + \cos t$

$\Rightarrow \frac{dt}{dy} = 2\cos^2\frac{t}{2}$

$\Rightarrow \frac{dt}{\cos^2\frac{t}{2}} = 2dy$

$\Rightarrow \int \sec^2\frac{t}{2}dt = 2\int dy$

$\Rightarrow 2\tan\frac{t}{2} = 2y + c$

$\Rightarrow 2\tan\frac{x+y}{2} = 2y + c$

$\Rightarrow \tan\frac{x+y}{2} = y + c$

46. If planes $\vec{r}.\left(p\hat{i} - \hat{j} + 2\hat{k}\right) + 3 = 0$ and $\vec{r}.\left(2\hat{i} - p\hat{j} - \hat{k}\right) - 5 = 0$ include angle $\frac{\pi}{3}$ then the value of p is

(A) 1, –3

(B) –1, 3

(C) –3

(D) 3

Answer (D)

Sol. $\cos 60^\circ = \frac{2p + p - 2}{\sqrt{p^2+5} \times \sqrt{p^2+5}}$

$\Rightarrow (3p - 2)2 = p^2 + 5$

$\Rightarrow p^2 - 6p + 9 = 0$

$\Rightarrow (p - 3)^2 = 0$

$\Rightarrow p = 3$

47. The order of the differential equation of all parabolas, whose latus rectum is $4a$ and axis parallel to the x-axis, is

(A) one

(B) four

(C) three

(D) two

Answer (D)

Sol. Required equation of parabola is

$(y - m)^2 = 4a(x - n)$

Here m and n are two independent arbitrary constant.

$\therefore$ Order of required differential equation is 2.

48. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $x - 3 = \frac{y-k}{2} = z$ intersect then the value of k is

(A) $\frac{9}{2}$ (B) $\frac{1}{2}$

(C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Answer (A)

Sol. General point on these lines are

$(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$ and $(\mu + 3, 2\mu + k, \mu)$

For point of intersection

$2\lambda + 1 = \mu + 3$ and $4\lambda + 1 = \mu$

$$\Rightarrow \lambda = -\frac{3}{2} \text{ and } \mu = -5$$

and $3\lambda - 1 = 2\mu + k \Rightarrow -\frac{9}{2} - 1 = -10 + k$

$$\Rightarrow 10 - \frac{11}{2} = k \Rightarrow k = \frac{9}{2}$$

49. If a line makes angles 120° and 60° with the positive directions of X and Z axes respectively then the angle made by the line with positive Y-axis is

(A) 150° (B) 60°

(C) 135° (D) 120°

Answer (C)

Sol. Let the angles which the line makes with positive directions of X, Y and Z axes be α, β and γ. Then

$\alpha = 120°$

$\gamma = 60°$

$\beta = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \frac{1}{4} + \frac{1}{4} + \cos^2\beta = 1$$

$$\Rightarrow \cos^2\beta = \frac{1}{2} \Rightarrow \cos\beta = \frac{1}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}}$$

Hence $\beta = 45°$ or 135°

50. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$ respectively. The position vector of the point N which divides the line segment LM in the ratio 2 : 1 externally is

(A) $3\vec{b}$ (B) $4\vec{b}$

(C) $5\vec{b}$ (D) $3\vec{a} + 4\vec{b}$

Answer (C)

Sol. Let the position vector of point N be $\vec{n}$. Then

$$\vec{n} = \frac{2(\vec{a} + 2\vec{b}) - 1(2\vec{a} - \vec{b})}{2 - 1}$$

$$= 5\vec{b}$$

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